Radiative energy transfer

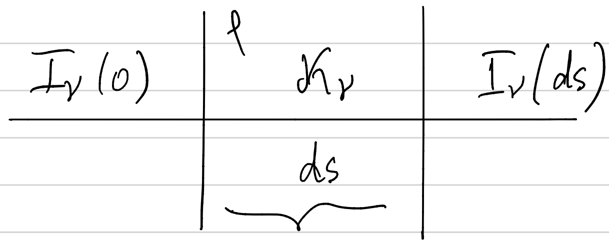
Radiative Energy Transfer

Extinction coefficient

When a radiation pass through a medium, it can be attenuated in proportion of I_ν and column density $\rho\, ds$

Consider $I_\nu(0)$ going through ds of density ρ

$\kappa_\nu \equiv$ opacity



$[\kappa_\nu] \equiv cm^2 g^{-1}$

The amount of absorbed radiation : $\boxed{dI_\nu = -\kappa_\nu \rho I_\nu\, ds}$

We shall see that κ_ν results from different physical processes (wait for next chapters ...)

$$\frac{dI_\nu}{ds} = -\kappa_\nu \rho I_\nu$$

if κ_ν et ρ are constants : $E_\nu(s) : I_\nu(s_0)\, e^{-\kappa_\nu \rho (s - s_0)}$

Mean free path :

$$\ell = \frac{1}{\kappa\rho} \qquad I_\nu(s) = I_\nu(s_0)\, e^{-\frac{(s-s_0)}{\ell}}$$

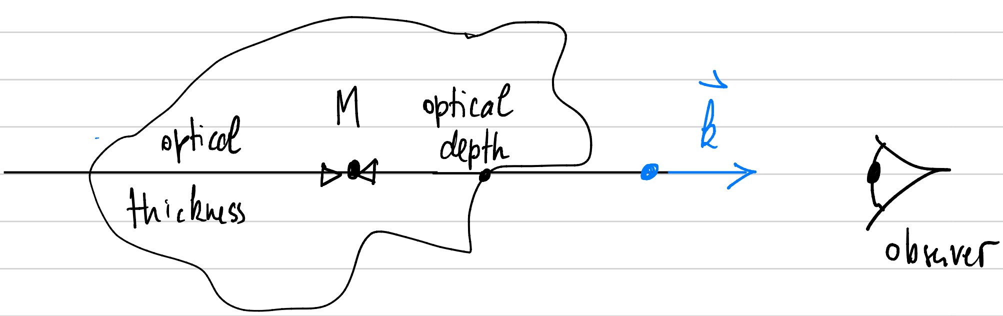
stellar interiors : $0.01 \lesssim \ell \lesssim 1$ cm.

After travelling a length ℓ, the intensity is decreased by a factor e.

1 μm	at	the center of $\odot$
1 cm	at	$T = 25000$ K
1 km	at	$T = 10\,000$ K
100 km	at	$T = 6000$ K
10^3 km	at	$T = 5000$ K red giants

Optical depth and stellar thickness

optical depth $\quad \tau_\nu = \int_{s_0}^{s} \kappa_\nu \rho \, ds' \longrightarrow I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$



extinction include a part of absorption and a part of scattering

$$K_\nu^{tot} = K_\nu^{scat} + K_\nu^{abs}$$

For a radiation with incident angle θ

Absorbed energy $$dU_\nu^{abs} = |dI_\nu| \, d\nu \, d\sigma \cos\theta \, d\Omega \, dt$$

$$= K_\nu \rho I_\nu \, ds \, d\nu \, d\sigma \cos\theta \, d\Omega \, dt$$

$\rho \, ds \, d\sigma$ = mass element = dm [light crossing dm]

$$dU_\nu^{abs} = K_\nu I_\nu \, d\nu \, dm \, d\Omega \, dt$$

Scattered light scattered in direction θ'

$$dU_\nu^{scat} = K_\nu^{scat} I_\nu \, dm \, d\nu \, d\Omega \, dt \left[\frac{d\Omega'}{4\pi} p^{diff}(\cos\theta') \right]$$

p^{diff} = phase function such that $\displaystyle\int_{\Omega'} \frac{p^{diff}}{4\pi} d\Omega' = 1$

Mie phase function $\quad p = \frac{3}{4}(1 + \cos^2\theta)$

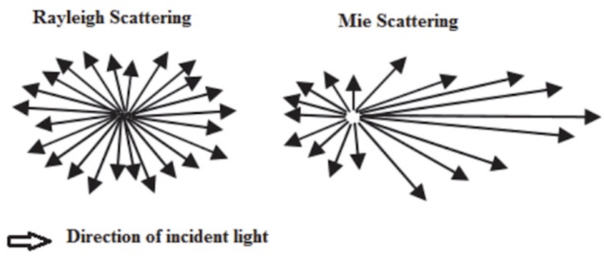
isotropic scattering $\quad p = 1$

Les mécanismes d'*absorption* et de *diffusion* sont décrits par les *théories de Mie et de Rayleigh*.

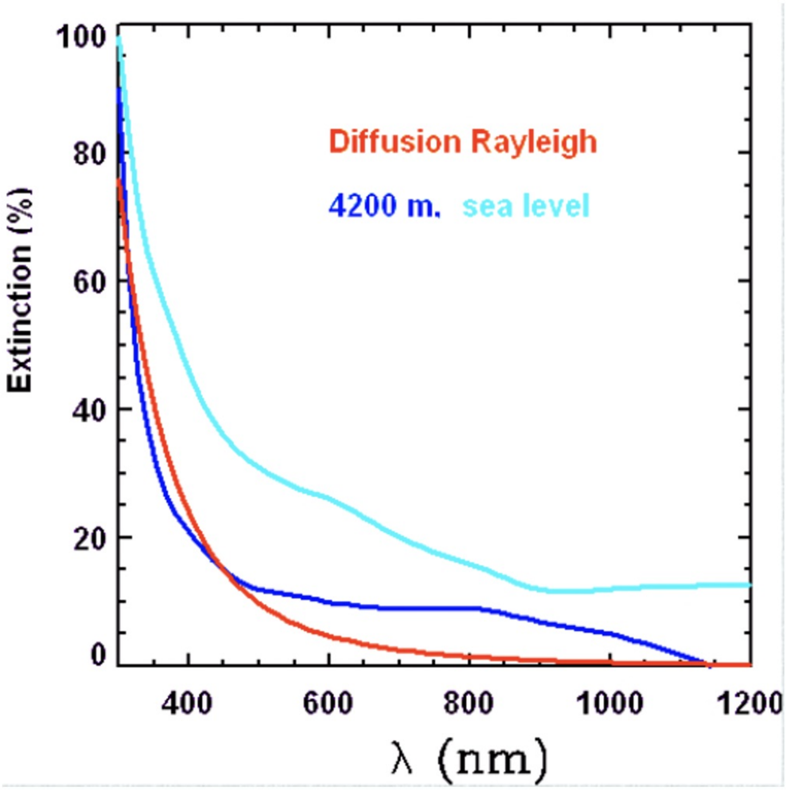
Ces deux théories sont une application des équations de Maxwell à des particules sphériques ou ellipsoïdales de petites tailles.

La *théorie de Rayleigh* s'applique à des particules d'un diamètre très inférieur à la longueur d'onde.

Lorsque le diamètre des particules est $\geq 0.1\boldsymbol{\lambda}$, le phénomène de diffusion est beaucoup plus complexe et il faut utiliser la *théorie de Mie*.

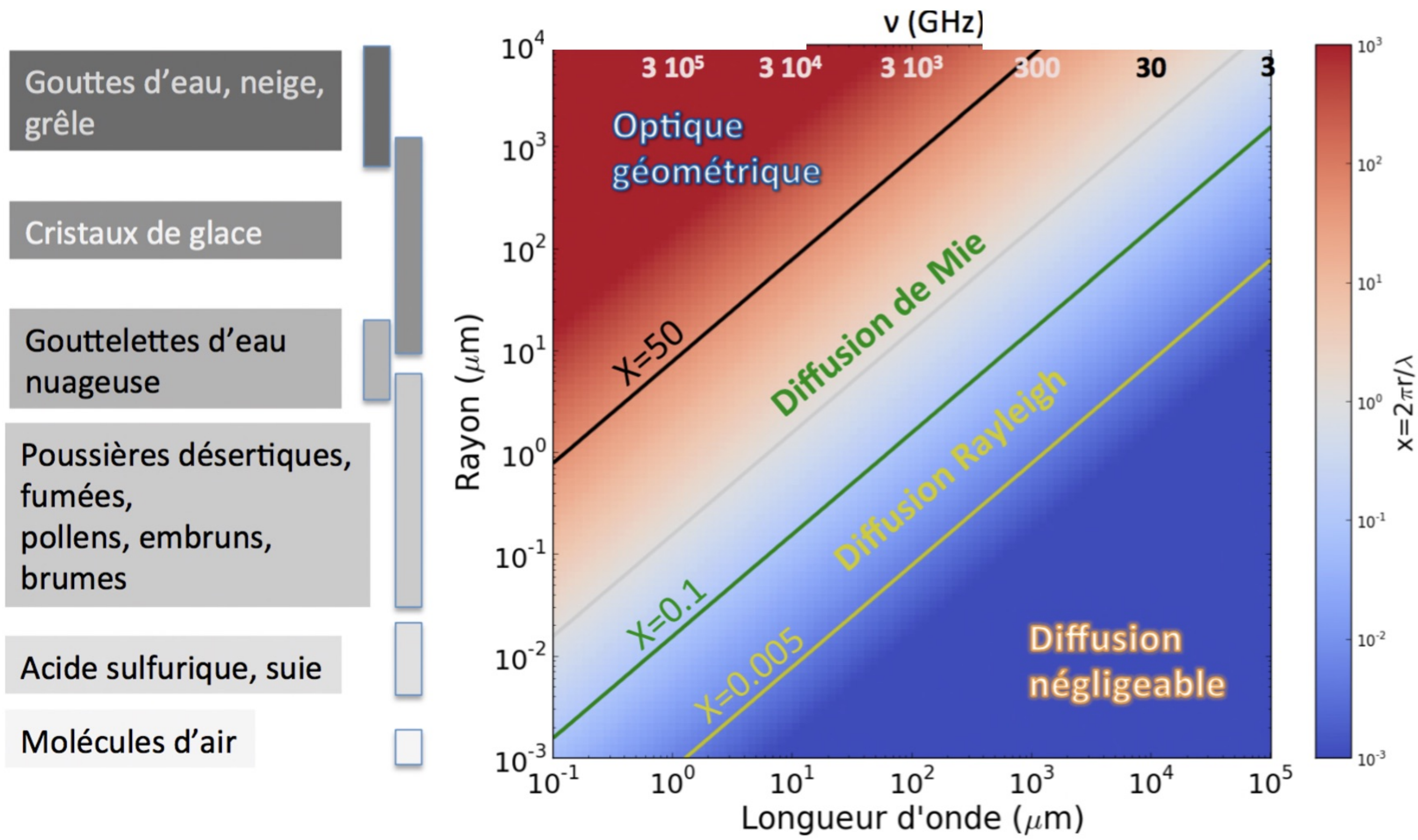


La diffusion Rayleigh varie comme $\boldsymbol{\lambda}^{-4}$
Elle est bien plus forte dans le bleu, à 400 nm, que dans le rouge à 650 nm. Ceci explique pourquoi le ciel est bleu la journée



Paramètre de Mie et diffusion

Paramètre de Mie:
$x = 2\pi r/\lambda$



visual domain

Albedo : $a_\nu = \dfrac{K_\nu^{diff}}{K_\nu^{diff} + K_\nu^{abs}}$

$$\Rightarrow dU_\nu^{diff} = \frac{U_\nu^{diff}}{U_\nu^{diff} + U_\nu^{abs}}$$

Absorption $\equiv$ conversion of radiative energy in thermal energy

Scattering $\equiv$ no conversion of energy, photons changes direction

Emission coefficient

A mass element can emit light (photons) per production of nuclear energy or as a black body (temperature)

$$dU_\nu^{em} = j_\nu \, d\nu \, dm \, d\Omega \, dt$$

$$[j_\nu] \equiv W \, kg^{-1} \, steradian^{-1} \, Hz^{-1}$$

one can write $dI_\nu^{em} = j_\nu \rho \, ds$

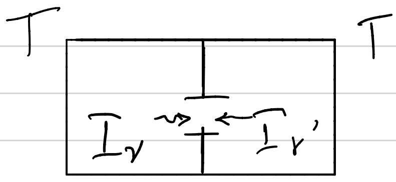
in analogy with $dI_\nu^{abs} = -K_\nu (I_\nu) \rho \, ds$

The Kirchhoff law

We consider a closed cavity, with walls at temperature T

The system is at thermal equilibrium

A each frequency there is equilibrium between emitted and absorbed energy



si le système est à ,
ce qu'il finira pas.
isolé, le rayonnement émis =
le rayonnement reçu.

||

irreversible transformation

→ La 1ère loi de Kirchhoff ne peut être qu'une fonction dépendante de la température.

Equilibrium between emission and absorption

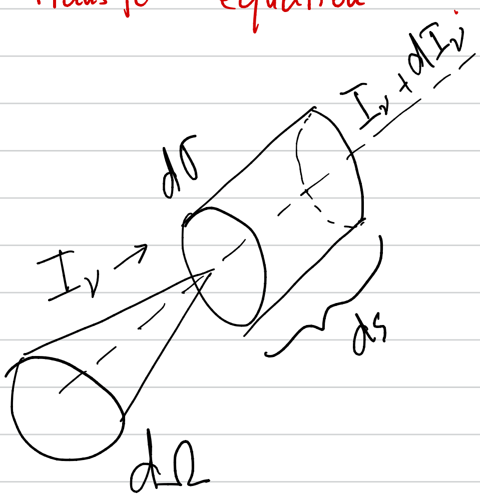
$$dU_\nu^{abs} = K_\nu I_\nu \, d\nu \, dm \, d\Omega \, dt$$

$$dU_\nu^{em} = j_\nu \, d\nu \, dm \, d\Omega \, dt$$

$$\Longrightarrow K_\nu I_\nu = j_\nu \qquad I_\nu = \frac{j_\nu}{K_\nu}$$

independant of medium

Transfer equation



In all celestial bodies, there is transfert of from the center to the surface irrespective of the source of internal energy

ex: gravitational collapse (proto $*$)

fusion ($*$), fission (earth)

Variation of energy in dt : $dU_\nu = dI_\nu \, d\sigma \, d\nu \, d\Omega \, dt$

Emitted energy $dU_\nu^{em} = j_\nu \, \rho \, d\sigma \, ds \, d\nu \, d\Omega \, dt$

Absorbed energy $dU_\nu^{abs} = K_\nu I_\nu \, \rho \, d\sigma \, ds \, d\nu \, d\Omega \, dt$

Balance : $dU_\nu = dU_\nu^{em} - dU_\nu^{abs}$

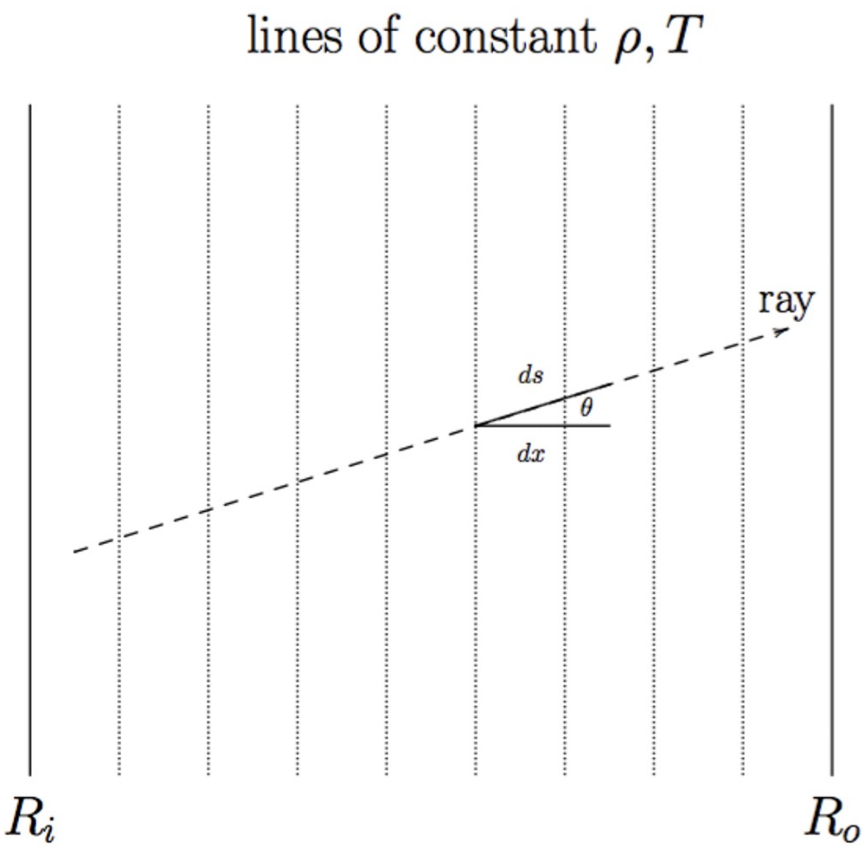
$$\frac{dI_\nu}{ds} = \rho \left(j_\nu - K_\nu I_\nu \right)$$

Equation of radiative transfert

Spherical geometry :

$$\frac{dP_{rad}}{dr} + \frac{1}{r}\left(3P_{rad} - u\right) + \frac{\rho K F}{c} = 0$$

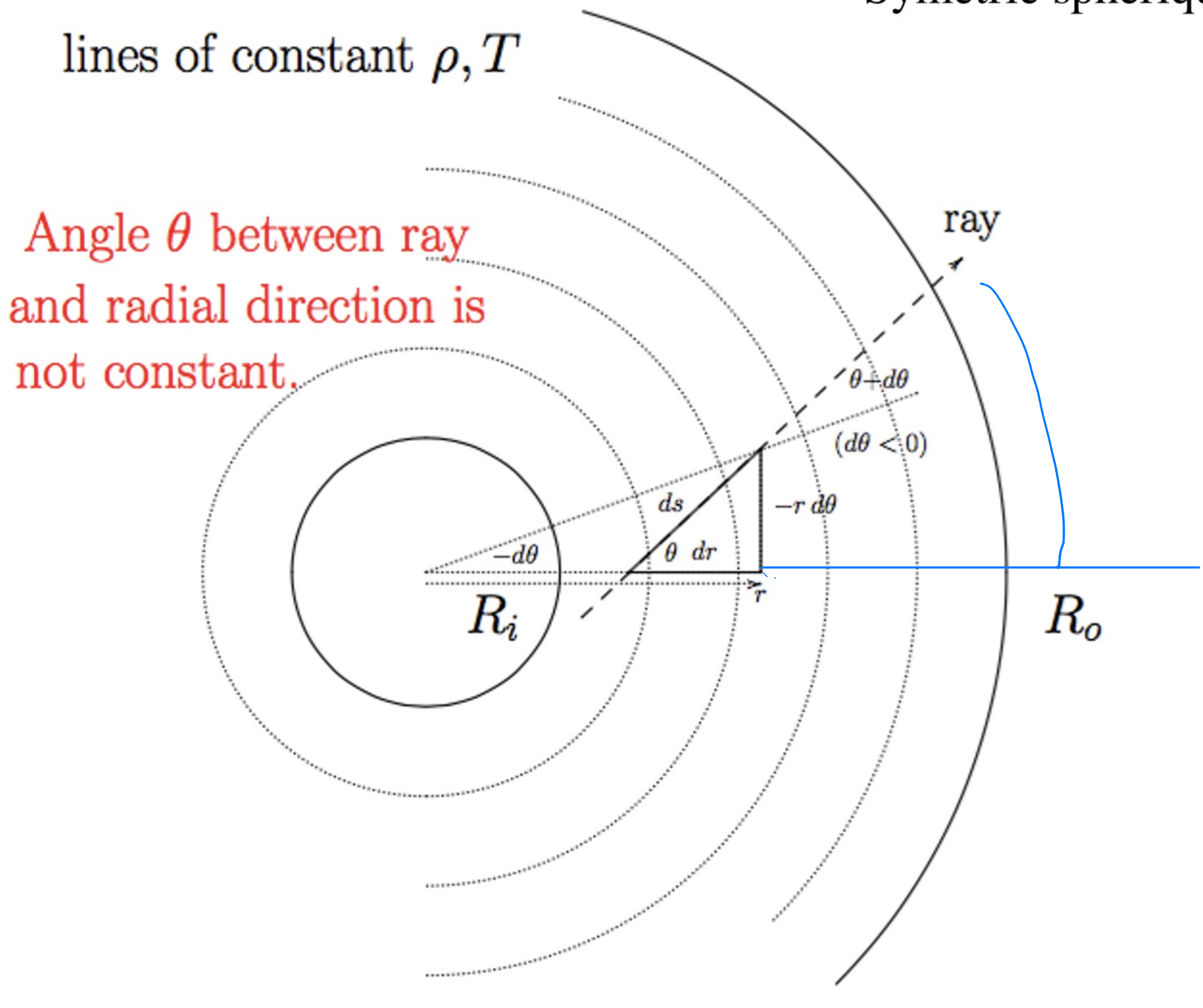
Plan parallèle



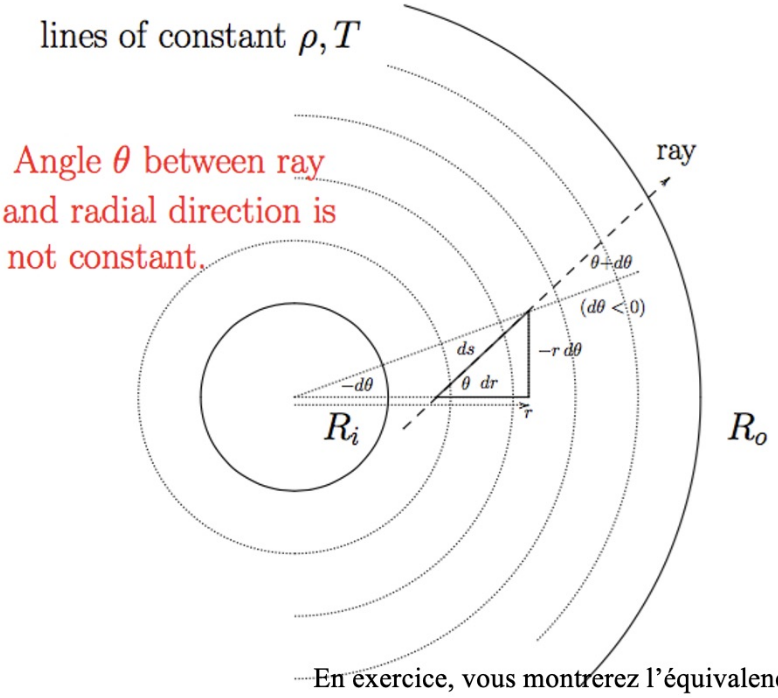
$$dx = \cos\theta \, ds = \mu \, ds$$

$$\frac{d}{ds} = \mu \frac{d}{dx}$$

Symétrie sphérique



Symétrie sphérique



$$\frac{d}{ds} = \frac{dr}{ds}\frac{\partial}{\partial r} + \frac{d\theta}{ds}\frac{\partial}{\partial \theta}$$

$$dr = \cos\theta\, ds = \mu\, ds \qquad \frac{dr}{ds} = \cos\theta$$

$$-r\, d\theta = \sin\theta\, ds \qquad \frac{d\theta}{ds} = \frac{-\sin\theta}{r}$$

En exercice, vous montrerez l'équivalence avec

$$\frac{\mathrm{d}P_{\mathrm{rad}}}{\mathrm{d}r} + \frac{1}{r}(3P_{\mathrm{rad}} - u) + \frac{\rho\kappa F}{c} = 0$$

Transfer equation in stellar interiors

LTE $\equiv$ Local Thermodynamic Equilibrium

$\equiv$ At each depth, the radiation field only depends on local T $\qquad I_\nu = B_\nu(T)$

LTE is satisfied if ℓ, photon mean free path $\ll$ scale of temperature variation

$$\frac{dT}{dr} \ll \frac{T}{\ell} \equiv \ell \ll \frac{T}{\frac{dT}{dr}}$$

- How much is $\frac{dT}{dr}$? $\quad \left|\frac{dT}{dr}\right| \sim \frac{T_c}{R} \sim \frac{10^7 K}{7.10^{10} cm} \sim 10^{-4}\, K.cm^{-1}$

if $T \sim 10\,000 K$ $\quad \frac{T}{\left|\frac{dT}{dr}\right|} \sim 10^8\, cm \sim 1000\, km.$

- How much is ℓ ? $\quad$ For $\rho = \langle \rho_\odot \rangle = 1.4\, g.cm^{-3}$

et $\kappa = 1 - 10\, cm^2 g^{-1}$, we have $\ell = \frac{1}{\kappa\rho} \sim 0.1 - 1\, cm$

LTE is verified $\quad \ell \ll \frac{T}{\frac{dT}{dr}}$ $\quad$ provided one is deep enough in the atmosphere

(Indeed @ $T = 6000K$ $\ell \gtrsim 1000\, km$!)

Stellar interiors : one can show that anisotropy is very weak

- let's develop I in power of $\cos\theta$

$$I(\theta) = \underbrace{I_0}_{\text{isotropy}} + \underbrace{I_1 \cos\theta}_{\text{deviation from isotropy}} + \underbrace{I_2 \cos^2\theta + \dots}_{\text{negligible}}$$

- Equation of transfer for a plane-parallel atmosphere (negligible curvature)

$$ds = \frac{dr}{\cos\theta}$$

$$\cos\theta \frac{dI_\nu}{dr} = \rho(j_\nu - \kappa_\nu I_\nu)$$

Replacing I by its development, one shows:

(will be done during exercices)

$$\frac{dI_{n-1}}{dr} = -\kappa\rho I_n \quad \text{for} \quad n > 0$$

$$\Rightarrow \frac{I_{n-1}}{R} \sim \kappa\rho I_n \Rightarrow \frac{I_n}{I_{n-1}} \sim \frac{1}{R\kappa\rho} \sim 10^{-10}$$

The series converges rapidely

$$I = I_0 + I_1 \cos\theta$$

Reminders

- $$dF_\nu = \frac{dU_\nu}{d\sigma\, d\nu\, dt} = I_\nu \cos\theta\, d\Omega$$

total flux at ν : $$F_\nu = \int_\Omega I_\nu \cos\theta\, d\Omega$$

- $$u_\nu = \frac{1}{c}\int_\Omega I_\nu\, d\Omega$$ (volume $= c\,dt\, d\sigma \cos\theta$)

- In case of symetry vs $\frac{\pi}{2}$ (stellar interiors and weak anisotropy)

$$P_{rad} = \frac{1}{c}\int_0^{2\pi}\int_0^{\pi} I \cos^2\theta\, d\Omega$$

Black body : $P_{rad} = \frac{1}{3} u = \frac{a}{3} T^4$ $\quad a = 7.565\ 10^{-15}\ \mathrm{erg/cm^3/K}$

$$\begin{cases} F_\nu = \int_\Omega \left[I_0 + I_1 \cos\theta \right] \cos\theta\, d\Omega \\ P_{rad} = \frac{1}{c}\int_0^{2\pi}\int_0^{\pi} \left[I_0 + I_1 \cos\theta \right] \cos^2\theta\, d\Omega \end{cases}$$

with $$\frac{dP_{rad}}{dr} + \frac{1}{r}\left(3P_{rad} - u\right) + \frac{\rho \kappa F}{c} = 0$$

- P_{rad} and u only depends on I_0 (modulo I_2) because the term in I_1 would given an odd integral (*). over a symetric interval

- F depend on I_1, however remains very small with respect to the thermal content $\frac{F}{uc} << 1$

- In the limits of these hypotheses, one can write

$$\frac{dP_{rad}}{dr} = -\rho\frac{\kappa F}{c}$$

- LTE $\Rightarrow$ $P_{rad} = \frac{1}{3}aT^4 \Rightarrow \frac{4}{3}aT^3\frac{dT}{dr} = -\rho\frac{\kappa F}{c}$

$$F = -\frac{4}{3}\frac{ac}{\rho\kappa}T^3\frac{dT}{dr}$$

Spherical symetry : $F = \frac{L_r}{4\pi r^2} = -C_{rad}\frac{dT}{dr}$

with $C_{rad} = \frac{4}{3}\frac{ac}{\rho\kappa}T^3 \propto \ell T^3$

NB : if $\frac{dT}{dr} = 0$ $\quad F_{rad} = 0$

So a star shines because there is a gradient in temperature (negative) whatever the origin of the gradient.

(*) $\int \cos^3\theta = \left[\sin\theta - \frac{\sin^3\theta}{3}\right]$

Mean Opacity

We have shown : $\frac{dP_{rad}}{dr}(\lambda) = -\frac{\kappa_\lambda \rho F_\lambda}{c}$

LTE : $P_{rad}(\lambda) = \frac{4\pi}{3c} B_\lambda(T)$.

Isotropy – relativistic
$P = \frac{1}{3}u$; $u = \frac{4\pi}{c} I_0$

$$\Rightarrow F_\lambda = -\frac{4\pi}{3\kappa_\lambda \rho} \frac{dB_\lambda}{dr}(T)$$

- We now try to express κ such that the transfer equation can be written with bolometric quantities

$$\frac{dP_{rad}}{dr} = -\kappa \frac{\rho F}{c} \Rightarrow F = -\frac{4\pi}{3\kappa\rho} \frac{dB(T)}{dr}$$

- We have

$$F = \int_\lambda F_\lambda \, d\lambda = -\frac{4\pi}{3\rho} \int \frac{1}{\kappa_\lambda} \frac{dB_\lambda(T)}{dr} d\lambda \quad (a)$$

and

$$F = -\frac{4\pi}{3\kappa\rho} \frac{dB(T)}{dr} \quad (b)$$

$$\frac{dB_\lambda}{dr} = \frac{dB_\lambda}{dT} \frac{dT}{dr} \longrightarrow F = -\frac{4\pi}{3\rho} \int \frac{1}{\kappa_\lambda} \frac{dB}{dT} \frac{dT}{dr} d\lambda$$

$$\Rightarrow \underbrace{-\frac{4\pi}{3\rho} \frac{dT}{dr} \int \frac{1}{\kappa_\lambda} \frac{dB_\lambda}{dT} d\lambda}_{(a)} = \underbrace{-\frac{4\pi}{3\kappa\rho} \frac{dB}{dT}^{(b)} \frac{dT}{dr}}_{\text{our wish}}$$

$$\Rightarrow \quad \frac{1}{\kappa} = \frac{1}{\frac{dB}{dT}} \int_0^\infty \frac{1}{\kappa_\lambda} \frac{dB_\lambda}{dT} \, d\lambda$$

Rosseland mean opacity

Tables provide the mean rosseland opacity → with some decomposition
→ bb, bf, ff and induced emission
Slides ...

Mass - Luminosity Relation

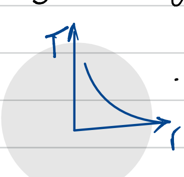
Ingredients

1. Transfer equation
2. Hydrostatic equilibrium
3. Ideal gas

(1) $$\frac{P_{rad}}{dr} = -\kappa \frac{\rho}{c} F_r \qquad P_{rad} = \frac{1}{3} a T^4 \qquad F_r = \frac{L_r}{4\pi r^2}$$

$$0 < r < R_*$$

$$\Rightarrow \quad \frac{1}{3} a \frac{d(T^4)}{dr} = -\frac{\kappa \rho}{c} F_r = -\frac{\kappa \rho}{c} \cdot \frac{L_r}{4\pi r^2}$$



approximation: $\frac{d(T^4)}{dr} \sim -\frac{T_c^4}{R_*} = -\frac{T^4}{R}$

considering: $r \sim \frac{R}{2}$

$$\Rightarrow \quad \frac{1}{3} a \left(- \frac{T^4}{R} \right) = - \frac{\kappa \rho}{c} F_r = - \frac{\kappa \rho}{c} \frac{4 L_r}{4 \pi R^2}$$

$$\Rightarrow \quad L_r = \frac{\pi a c}{3 \kappa \rho} R T^4 \qquad (1)$$

- How much is T^4 ?

Hydrostatic Equilibrium $\quad \frac{dP}{dr} = - \frac{GM}{R^2} \bar{\rho}$

$$\Rightarrow \quad P_c \simeq \frac{GM}{R} \bar{\rho} \qquad (2)$$

Ideal gas

$$P = \frac{\rho}{\mu m_H} k T$$

$\mu =$ mean molecular weight

$$P_c = \frac{\bar{\rho}}{\mu m_H} k T_c \qquad (3)$$

donc $\quad T_c = \frac{\mu m_H}{k \bar{\rho}} P_c$

$$T_c = \frac{\mu m_H}{k \cancel{\bar{\rho}}} \frac{GM}{R} \cancel{\bar{\rho}} \qquad \text{[including (2)]}$$

Taking back (1), introducing $T \equiv T_c$ and $\rho = \bar{\rho} = \frac{M}{\frac{4}{3} \pi R^3}$

Finally $$L = \frac{\pi a c}{9K} \frac{4\pi R^3}{M} R \left(\frac{\mu m_H}{k} \frac{GM}{R} \right)^4$$

$$\left(\frac{\pi^2}{9} \sim 1 \right)$$

Conclusions :

$$\boxed{L \simeq 4ac \left(\frac{G m_H}{k} \right)^4 \frac{\mu^4 M^3}{K}}$$

- $L \propto M^3$ if $M = 0.1 M_\odot$ $L \sim 10^{-3} L_\odot$

1	1
10	1000
100	10^6 !

- L depends on μ at M given, an He star is more luminous than a H star
- L depends on $\frac{1}{K}$ If $K \nearrow$ $L \searrow$
- The relation $M - L$ is independent of the production of energy.

Model of grey atmosphere in radiative equilibrium

Application de l'équation de transfert

Gray $\equiv$ κ independent of ν

We will build $T(\tau)$ in a simple and analytical way with 4 hypotheses.

1. Plan-parallel atmosphere (thin atmosphere)
2. LTE
   at each depth one source function : $S_\nu = \dfrac{j_\nu}{\kappa_\nu} = B_\nu(T)$ [Kirchhoff]
3. Radiatif equilibrium $\equiv$ constant flux
4. κ independent of ν

Equation of transfer $\dfrac{dP_{rad}}{dr} = -\kappa\rho\dfrac{F}{c}$ with $F = \sigma T_{eff}^4 = $ cst

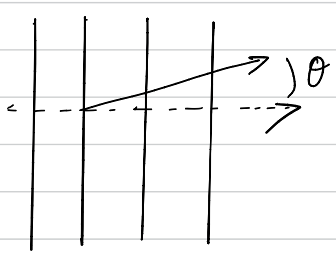
optical depth $d\tau = -\kappa\rho\, dr \Rightarrow \dfrac{dP_{rad}}{d\tau} = \dfrac{F}{c}$

⚠ $\tau(0) = 0$ at the surface $\Rightarrow$ $P_{rad} = \dfrac{F}{c}\tau + P_{rad}(0)$ (1)

What is $P_{rad}(0)$?

Eddington approximation : I is constant on the 2 hemisphere

"pp"



$$I(\mu) = \begin{cases} I_{out} \ ; \ \mu > 0 & \left(\theta < \frac{\pi}{2}\right) \\ I_{in} \ ; \ \mu < 0 & \left(\theta > \frac{\pi}{2}\right) \end{cases}$$

$\mu = \cos\theta$

$$P_{rad} = \frac{1}{c}\int_0^{2\pi}\int_0^{\pi} I \cos^2\theta \, d\Omega$$

$$\langle I \rangle = \frac{1}{2}\left(I_{out} + I_{in}\right) \qquad P_{rad} = \frac{2\pi}{c}\int_0^1 I \mu^2 d\mu .$$

$F_\nu = F_\nu^+ - F_\nu^-$ $\qquad F = \pi \langle I \rangle$

At the surface $I_{in} = 0$ $\Rightarrow$ $P_{rad} = \frac{2\pi}{c}\int_0^1 I_{out}\,\mu^2 d\mu = \frac{2\pi}{3c} I_{out}$

$F = \pi I_{out}$

$$\Rightarrow P_{rad} = \frac{2F}{3c} = P_{rad}(0) \qquad (2)$$

We know $P_{rad} = \frac{1}{3} a T^4$

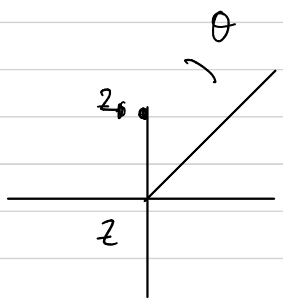
(1) et (2) $\rightarrow$ $\frac{1}{3} a T^4 = \frac{2F}{3c} + \frac{F}{c}\tau = \frac{F}{c}\left(\frac{2}{3} + \tau\right)$

$F = \sigma T_{eff}^4$ $\qquad \Rightarrow T^4 = \frac{3}{ac}\sigma T_{eff}^4\left(\tau + \frac{2}{3}\right)$

and $a = \frac{4\sigma}{c}$ $\Rightarrow$ $T^4 = \frac{3}{4} T_{eff}^4\left(\tau + \frac{2}{3}\right)$

Remarque. if $\tau = 0$ $\quad T(0) = \left(\frac{1}{2}\right)^{1/4} T_{eff} = 0.841\, T_{eff}$.

Outgoing / Emergent intensity



Solution of the transfer equation without approximation pp

$$\frac{dI_\nu}{ds} = j_\nu \rho - \kappa_\nu \rho I_\nu$$

$$\cos\theta \frac{dI_\nu}{dz} = j_\nu \rho - \kappa_\nu \rho I_\nu$$

reduced equation $$\mu \frac{dI_\nu}{dz} = j_\nu \rho - \kappa_\nu \rho I_\nu$$

source function $\quad S_\nu = \dfrac{j_\nu}{\kappa_\nu}$ and $d\tau_\nu = -\kappa_\nu \rho\, dz$

$$\Rightarrow \mu \frac{dI_\nu}{d\tau} = -\frac{1}{\kappa_\nu \rho}\left(j_\nu \rho - \kappa_\nu \rho I_\nu\right) = I_\nu - S_\nu$$

integral in 2 steps

(A) reduced equation $\quad \mu \dfrac{dI_\nu}{d\tau} = I_\nu \Rightarrow I_\nu = C e^{\frac{\tau}{\mu}}$

(B) method of the variation of constants: $C = C(\tau)$

$$\mu \left[\frac{dC}{d\tau} e^{\frac{\tau}{\mu}} + \frac{C}{\mu} e^{\frac{\tau}{\mu}}\right] = C e^{\frac{\tau}{\mu}} - S_\nu$$

$$\mu \frac{dC}{d\tau} e^{\frac{\tau}{\mu}} = -S_\nu \Rightarrow \frac{dC}{d\tau} = -\frac{S_\nu}{\mu} e^{-\frac{\tau}{\mu}}$$

$$C(\tau) = -\int_{\tau_1}^{\tau} \frac{S_\nu}{\mu} e^{-t/\mu} dt + C_1$$

with $C_1 = I(\tau_1) e^{-\frac{\tau_1}{\mu}}$

$$I(\tau,\mu) = \underbrace{I(\tau_1,\mu) e^{-\frac{\tau_1-\tau}{\mu}}}_{\substack{\text{attenuation} \\ \times \text{ incident radiation} \\ \text{at depth } \tau_1}} + \underbrace{\int_{\tau}^{\tau_1} S(t,\mu) e^{-\frac{t-\tau}{\mu}} \frac{dt}{\mu}}_{\substack{\text{Contribution of emission} \\ \text{along the path between } \tau_1 \text{ and } \tau}}$$

- The emergent intensity is equal to the sum of the contributions from the source functions of the deeper layers, each attenuated by an absorption coefficient.
- To calculate the emergent intensity, one must know $S(\tau_\nu)$, hence how the temperature, the pressure, the chemical composition vary with τ

$$I(\tau,\mu) = I(\tau_1,\mu)\, e^{-\frac{\tau_1 - t}{\mu}} + \int_{\tau}^{\tau_1} S(t,\mu)\, e^{-\frac{t-\tau}{\mu}} \frac{dt}{\mu}$$

(A) Semi-infinite medium $\tau_1 \to \infty$

$$I(\tau,\mu) = \int_{\tau}^{\infty} S(t,\mu)\, e^{-\frac{t-\tau}{\mu}} \frac{dt}{\mu}$$

Emergent intensity : $$I(0,\mu) = \int_0^{\infty} S(t,\mu)\, e^{-t/\mu} \frac{dt}{\mu}$$

(B) μ varies from 0 to 1, one can consider $\mu = 1$

$$I(\tau,1) = I(\tau_1,1)\, e^{-(\tau_1-\tau)} + \int_{\tau}^{\tau_1} S(\tau,1)\, e^{-(t-\tau)}\, dt$$

Homogeneous medium : $S = cste$ $\kappa_\nu = cste$

Emergent intensity : $I(0,1) = I(\tau_1) e^{-\tau_1} + S \underbrace{\int_0^{\tau_1} e^{-t} dt}_{1 - e^{-\tau_1}}$

$$I(0,1) = I(\tau_1) e^{-\tau_1} + S\left(1 - e^{-\tau_1}\right)$$

Optically thin medium $\tau_1 \ll 1$

$$e^{-\tau_1} \sim 1 - \tau_1$$

$$I(0) = \left[S - I(\tau_1)\right] \tau_1 + I(\tau_1)$$

- $I(0) > I(\tau_1)$ si $S > I(\tau_1)$ emission.
- $I(0) < I(\tau_1)$ si $S < I(\tau_1)$ absorption

si $I(\tau_1) = 0$ $I(0) = S\tau_1$.

Optically thick medium $\tau_1 \gg 1$

$$e^{-\tau_1} \sim 0 \Rightarrow I(0) \simeq S$$

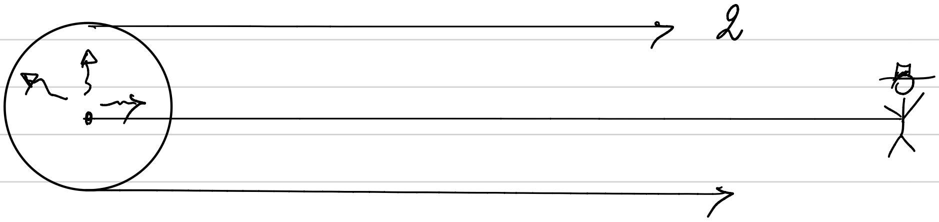
Stellar atmosphere = optically thin medium placed above an optically thick medium.

Assombrissement centre-bord.

At the surface of the Sun, flux is identical everywhere but the specific intensity depends on θ

For a remote observer $\theta = 0$ center of surface $(\mu = 1)$

$\theta = \frac{\pi}{2}$ at the edges $(\mu = 0)$



For a grey atmosphere and under LTE $(\kappa \neq f(\nu))$

$$I(0,\mu) = \int_0^\infty S(t,\mu)\, e^{-t/\mu} \frac{dt}{\mu}$$

LTE : $S = B = \frac{\sigma}{\pi} T^4 = \frac{3}{4\pi} \sigma T_{eff}^4 \left(\tau + \frac{2}{3}\right) = \frac{3}{4\pi} F \left(\tau + \frac{2}{3}\right)$

$$I(0,\mu) = \frac{3F}{4\pi} \Bigg[\mu \underbrace{\int_0^\infty \frac{\tau}{\mu} e^{-\frac{\tau}{\mu}} \frac{d\tau}{\mu}}_{\int_0^\infty x e^{-x} dx} + \frac{2}{3} \underbrace{\int_0^\infty e^{-t/\mu} \frac{dt}{\mu}}_{\int_0^\infty e^{-x} dx = -e^{-x}\big|_0^\infty = 1} \Bigg]$$

$$\int_0^\infty x e^{-x} dx \to u\,dv$$

$d(uv) = u\,dv + v\,du$

avec $u = x \quad dv = e^{-x}$

$$\underbrace{-x e^{-x}\Big|_0^\infty}_{0} + \underbrace{\int_0^\infty e^{-x} dx}_{1}$$

$$\Rightarrow \quad I(0,\mu) = \frac{3F}{4\pi}\left(\mu + \frac{2}{3}\right) \quad \text{et} \quad I(0,1) = \frac{3F}{4\pi} \cdot \frac{5}{3}$$

We look at the ratio $\dfrac{I(0,\mu)}{I(0,1)} = \dfrac{3}{5}\left(\mu + \dfrac{2}{3}\right)$

$$= 0.4 + 0.6\,\mu$$

si $\mu = 0$ $\quad$ 0.4 (edge)

$\mu = 1$ $\quad$ 1 (Centre)

see slides.